The Partial Solution of First Price Sealed Bid Auction in the Scope of Mechanism Design Theory and With Uniformly Distributed Types

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Կարգերի անընդհատ հավասարաչափ բաշխմամբ առաջին գնով գաղտնագրված աճուրդի մասնավոր լուծումը կառուցակարգերի նախագծման տեսության շրջանակներում

Պետրոսյան Ռաֆայել Մ.
ՀՀ պետական կառավարման ակադեմիայի, Կառավարման ամբիոն ասպիրանտ (Երևան, ՀՀ)

Ամփոփագիր. Հոդվածում վերլուծության է ենթարկվել առաջին գնով գաղտնագրված աճուրդը կառուցակարգերի նախագծման տեսության շրջանակներում և կարգերի անընդհատ հավասարաչափ բաշխման պայմաններում: Դիտարկվել է երկու անհատներից բաղկացած առաջին գնով գաղտնագրված աճուրդը՝ դուրս բերելով աճուրդի մասնվաոր լուծումը: Գնահատվել է աճուրդի Բայեսյան Նեշի ռազմավարությունները, հավասարակշռության վիճակը, կիրարկվող սոցիալական ընտրության ֆունկցիան: Երկու անհատից բաղկացած աճուրդի սոցիալական ընտրության ֆունկցիան ճշմարացիորեն կիրարկելու համար նախագծվել է համապատասխան կառուցակարգը: Գնահատվել են նաև մասնակցի հաղթելու հավանականության, սպասվող օգտակարության և վաճառողի սպասվող եկամտաբերության փոփոխականները:
Հանգուցաբառեր՝ Կառուցակարգերի նախագծման տեսություն, կառուցակարգ, աճուրդ, առաջին գնով գաղտնագրված աճուրդ, գերիշխող ռազմավարություն, Բայեսյան Նեշի հավասարակշռության վիճակ, խրախուսական համատեղելիություն, սոցիալական ընտրության ֆունկցիա

Частичное решение закрытого аукциона первой цены в области теории дизайна механизма и с равно распределенными типами

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Аннотация. В статье анализируется аукцион первый цены в рамках теории конструктирования механизмов и с равномерно распределенными типами. Было найдено частичное решение для аукциона первой цены, состоящий из двух агентов. Оценены стратегии байесовского равновесия Нэша, состояние равновесия и реализуемая функция социального выбора для аукциона. Механизм был разработан для правдивой реализации функции социального выбора для аукциона, состоящего из двух агентов. Были оценены такие переменные, как вероятность победы агента, ожидаемая полезность агента и ожидаемый доход продавца.
Ключевые слова: Теория дизайна механизма, механизм, аукцион, аукцион с закрытыми предложениями по первой цене, доминирующая стратегия, состояние байесовского равновесия по Нэшу, совместимость стимулов, функция социального выбора

Introduction

Mechanism design theory has widespread applications across various fields of economics. The exploration of mechanism design and its market implementations has attracted considerable attention from researchers. The primary aim of mechanism design theory is to develop mechanisms that achieve pre-defined objectives.

Mechanism design operates within the constraints of informational asymmetry and individual rationality. In this context, agents act in their self-interest, striving to maximize their utility. Meanwhile, the mechanism designer aims to achieve desirable social and economic outcomes for all parties, guided by specific criteria of interest [**1**, pp. 1-19], [**2**, pp. 1-8].

Particularly notable is the theory's efficiency in auction theory. Mechanism design theory facilitates the analysis and comparison of different auction types, assessing the efficiency of their implementation and application. Leveraging this theory, new auction models have been developed, whose equilibrium states align with the mechanism designer's initial goal function [**3**, pp. 1-62]:

The goal of this article is to analyse the first price sealed bid auction consisting of two agents within the scope of mechanism design theory.

In order to achive the goal the following objectives will be addressed:

- analyse the first price sealed bid auction consisting of two players using the methodology and tools provided by mechanism design
- identify the Bayesian Nash equilibrium strategies of the agents and the Bayesian Nash equilibrium state of the auction,
- discover the social choice function that is implemented by the mechanism of first price sealed bid auction,
- assess the winning probability of agents implementing Bayesian Nash equilibrium strategies based on types,
- calculate expected utilities of agents and the expected revenue of the seller.

The object of the article is the first price sealed bid auction consisting of two agents, and the subject of the article are the problems of design and implementation of the first price sealed bid auction.

Methodology

First price sealed bid auction has been analyzed in the article. During first price sealed bid auction the participants of the auction make their bids simultaneously. The product is sold to the agent, who made the highest bid, and the agents pays the bided amount. [**4**, pp. 421-443], [**5**, pp. 888-907]. First price sealed bid auction with one indivisible good is considered in the article.

Within the mechanism design theory the term θ_i represents the type of ith agent. With the use of an economic mechanism several agents make a collective decision, but prior to decision-making each agent privately observes a distinct parameter or "message", which delineates his preferences and, consequently influences his utility function. Mathematically, this concept is articulated by incorporating the parameter Θ_i, which is exclusively observed by agent.

The inclusion of θ_i in the utility function $u_i(a, \theta_i)$, signifies the idea, that an agent's type directly influences his preferences and utility function. In case of first price sealed bid auction the type θ_i indicates the willingness to pay of the agent for the product, while the Θ_i is the set of all possible types [**6**, pp. 857-897].

$f: \Theta_1 \times \Theta_2 \times \ldots \times \Theta_N \to X$ social choice function defines an alternative $f(\theta) = x \in X$ for each type vector $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$. Social choice function is ex post efficient or Pareto efficient, if for any type vector $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$ there is no alternative $x \in X$, where $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i)$ for each agent i, and $u_i(x, \theta_i) > u_i(f(\theta), \theta_i)$ for some agent. The social choice function of the first price sealed bid auction is Pareto efficient, if the product is sold to the agent with the highest valuation.

Mechanism $\Gamma = (S_1, S_2, \ldots, S_N, g(\cdot))$ is a set of N strategy sets $(S_1, S_2, \ldots, S_N)$ and $g: S_1 \times S_2 \times \cdots \times S_N \to X$ decision function: Each agent i observes his θ_i type and based on S_i strategy sends a message to the mechanism, which makes a collective decision based on $g(\cdot)$ decision function and chooses an alternative x from the set of alternatives X. In first price sealed bid auction all agents privately observe their own types and make bids based on some strategy. According to the decision function the product is sold to the agent, who places the highest bid$(y_i(b) = 1 \; if \; b_i = \max\{b_1, b_2, \cdots, b_N\}, 0` \; otherwise)$, where b_i is the bid placed by the ith agent, and b is the vector of placed bids.

In case of an auction the alternative also consists of t_i payments, which align the decision function of the mechanism with the highest placed bid $t_i = -b_i \times y_i(b)$: The mathematical description of the auction as a mechanism is given bellow:

$$(1) \quad \Gamma = (S_1, S_2, \ldots, S_N, g(\cdot))$$

$$(2) \quad s_i \in S_i \in R_+$$

$$(3) \quad g(b) = x \in X = \begin{bmatrix} y_i(b) = 1 \; \text{եթե} \; b_i = \max\{b_1, b_2, \cdots, b_N\} \\ y_i(b) = 0 \; \text{եթե} \; b_i \neq \max\{b_1, b_2, \cdots, b_N\} \\ t_i = -b_i \times y_i(b) \end{bmatrix}$$

Mechanism $\Gamma = (S_1, S_2, \ldots, S_N, g(\cdot))$ implements the social choice function, if there exists a strategy vector $(s_1^*, s_2^*, \ldots, s_N^*)$, which results in equilibrium state for the mechanism Γ (Nash equilibrium, Bayesian Nash equilibrium, etc.) [**7**, pp. 75-313], where the decision function and the social choice function are equal $g(s_1^*(\theta_1), s_2^*(\theta_2), \ldots, s_N^*(\theta_N)) = f(\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$.

In essence first price sealed bid auction is a direct mechanism, as the agents provide information regarding their types (willingness to pay). The social choice function implemented by first price sealed bid auction is given in the analysis part of the article.

Social choice function f is truthfully implementable (incentive compatible) [**8**, pp. 76-93], if the direct mechanism $\Gamma = (\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot))$ there exists a vector of strategies $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ which results in an equilibrium state, where $s_i^*(\theta_i) = \theta_i$ for all $\theta_i \in \Theta_i$ and i. Therefore, the social choice function is truthfully implementable, if the strategy of thrutful revelation of types results in an equilibrium state of the mechanism $\Gamma = (\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot))$. The evaluation of truthful implementation or incentive compatibility of the first price sealed bid auction is given in the analysis part of the article.

In case of first price sealed bid auction the strategies of the agents depend on expectations of the strategies of other agents, therefore the solution to the problem (the vector of equilibrium strategies) cannot be dominant strategy Nash equilibrium, but a Bayesian Nash equilibrium. [**9**, pp. 85-102]. In case of first price sealed bid auction rational agents try to maximize the expected utility, and as the expected utility is highest in case of Bayesian Nash equilibrium strategies, then this is the strategy to be implemented. The vector of strategies $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ is considered to be Bayesian Nash equilibrium strategy for the mechanism $\Gamma = (S_1, S_2, \dots, S_N, g(\cdot))$, if for all agents i and all types θ_i the expected utility resulting from implementation of Bayesian Nash equilibrium strategy is higher than the expected utility resulting from any other strategies [**10**, pp. 296-334].

(4) $$E_{\theta_{-i}}[u_i(g(s_i^*(\theta_i), s_{-i}^*), \theta_i)|\theta_i] \geq E_{\theta_{-i}}\left[u_i\left(g\left(s_i^{'}(\theta_\mathrm{i}), s_{-i}^*\right), \theta_\mathrm{i}\right)\middle|\theta_i\right]$$
$$for\ \forall\ i, \theta_i, s^{'}, s_{-i}$$

Mechanism $\Gamma = (S_1, S_2, \dots, S_N, g(\cdot))$ implements social choice function f with a Bayesian Nash equilibrium strategy, if for the mechanism Γ there exists Bayesian Nash equilibrium strategy vector $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$, which results to an equilibrium state for the mechanism, where the decision function is equal to the social choice function $g(s^*(\theta)) = f(\theta)$, for all $\theta \in \Theta$. If there exist a Bayesian Nash equilibrium strategy for the first price sealed bid auction, then it implements some social choice function.

The social choice function is truthfully implementable with Bayesian Nash equilibrium strategy (Bayesian Nash incentive compatible), if the strategy $s^*(\theta) = \theta$ is Bayesian Nash equilibrium strategy for the mechanism $\Gamma = (\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot))$. According to the revelation principle, if there exists a mechanism $\Gamma = (S_1, S_2, \dots, S_N, g(\cdot))$, which implements a social choice function f in Bayesian Nash equilibrium, then the social choice function f is truthfully implementable in Bayesian Nash equilibrium.

Analysis

In case of first price sealed bid auction the buyers place their bids simultaneously. The product is sold to the agent, who placed the highest bid, and the agent pays his bid. In the scope of the analysis first price sealed bid auction with one indivisible good is considered.

The set of $I = \{1,2\}$ agents is considered. The set of mutually exclusive alternatives X is considered $x \in X$. The set of alternatives for the first price sealed bid auction is given bellow:

(5) $$X = \{(y_1, y_2, t_1, t_2) \colon y_i = \{0,1\}\ \text{և}\ t_i \in R\ \text{բոլոր}\ i\ \text{համար}, \textstyle\sum_i y_i = 1, \sum_i t_i \leq 0)\ \}$$

The alternative x consists of y_i variable, which is 1 or 0 for each agent i. If $y_i = 1$, then the product is bought by ith agent, and if $y_i = 0$, the product is not sold to ith agent: The condition $\sum_i y_i = 1$ ensures, that the product is sold to only one agent. The alternative x also consists of t_i variable, which shows the payment of ith agent (the payments have negative sign: $\sum_i t_i \leq 0$): Therefore, in case of first price sealed bid auction the alternative x shows to whom the product is sold and how much money is paid by each agent [**11**, pp. 1-30].

The payments t_i for first price sealed bid auction are given in the formula bellow:

(6) $t_i = b_i y_i$,

where the b_i is the placed bid by the agent. If the agent wins the auction, then he pays his bid $y_i = 1$ and $t_i = b_i \times 1 = b_i$. Meanwile, if the agent does not win the auction, he does not pay anything $y_i = 0$, իսկ $t_i = b_i \times 0 = 0$:

The utility function $u_i(a, \theta_i)$ of the agent depends on θ_i parameter and the result a of the auction (to whom the product is sold and how much is paid for the product). In case of first price sealed bid auction the type θ_i shows the willingness to pay of the agent. The utility function of the agents in the first price sealed bid auction is given bellow:

(7) $$u_i(a, \theta_i) = \theta_i y_i - t_i = \theta_i y_i - b_i y_i$$

The type of ith agent is represented by θ_i, and the set of possible types by Θ_i. θ is the vector of types of all agents $\theta = (\theta_1, \theta_2, \theta_3, \dots, \theta_N)$. It is assumed that types θ_i are random variables, which have a uniform distribution [**12**, pp. 1-85], [**13**, pp. 176-182]: θ_i types are normalized within the range of [0,1]. The probability distribution function and cumulative distribution function of uniform distribution are given bellow:

$$(8)\quad f(\theta_i) = \begin{cases} \frac{1}{b-a} = \frac{1}{1-0} = 0.04, if\ 0 \le \theta_i \le 1 \\ 0, if\ 1 \ge \theta_i\ or\ 1 \le \theta_i \end{cases}$$

$$(9)\quad F(\theta_i) = \begin{cases} 0, if\ 1 \ge \theta_i \\ \frac{\theta_i - 0}{1-\theta_i}, if\ 0 \le \theta_i \le 1 \\ 1, if\ 1 \le \theta_i \end{cases}$$

First price sealed bid auction is represented by the mechanism $\Gamma = (S_1, S_2, g(\cdot))$. As the set of agents consist of two agents, the set of strategies also consist of two objects $\Gamma = (b_1(\theta_i), b_2(\theta_i), g(\cdot))$. The ith agent observes his type θ_i (known only to him), and according to b_i function places a bid b.

The objective of the ith agent in first price sealed bid auction is to maximize his expected utility by choosing a strategy function b_i:

(10) $$\max((\theta_i - b_i(\theta_i))F(b_i(\theta_i))$$

(11) $$F(b_i(\theta_i) = Prob(b_i \ge b_{-i})$$

(12) $$\max((\theta_1 - b_1(\theta_1))F(b_1(\theta_1))$$

(13) $$\max((\theta_2 - b_2(\theta_2))F(b_2(\theta_2))$$

The ith agent chooses such a bid function b_i, which maximises the product of his utility and probability of winning. In this case the problem of the agent can be solved by first order differentiation.

$$(14)\quad \max((\theta_i - b_i(\theta_i))F(b_i(\theta_i)) = \max\left((\theta_i - b_i(\theta_i))\frac{b_i(\theta_i)-0}{1-0}\right)$$

$$(15)\quad (\theta_i b_i(\theta_i) - b_i(\theta_i) b_i(\theta_i))' = (\theta_i b_i(\theta_i) - b_i(\theta_i)^2)' = \theta_i - 2b_i(\theta_i) = 0$$

$$(16)\quad \theta_i - 2b_i(\theta_i) = 0 \leftrightarrow b_i(\theta_i) = \frac{\theta_i}{2}$$

According to the analysis above, it can be stated, that the optimal strategy of the agent, which maximizes his expected utility is the strategy $\frac{\theta_i}{2}$. Therefore, according to the optimal strategy the agent having θ_i type places a bid which is the half of his type. As the abovementioned strategy is optimal strategy for both of the agents, the implementation of the optimal strategies results in equilibrium state. Meanwhile, by the description of the equilibrium state the social choice function implemented by the mechanism of the auction can be revealed. The implementation of Bayesian Nash equilibrium strategies results in the following equilibrium state.

$$(17)\quad y_1(\theta) = 1\ \text{եթե}\ \theta_1 \ge \theta_2\ ;\ y_1(\theta) = 0\ \text{եթե}\ \theta_1 < \theta_2$$
$$y_2(\theta) = 1\ \text{եթե}\ \theta_2 > \theta_2\ ;\ y_1(\theta) = 0\ \text{եթե}\ \theta_2 \le \theta_1$$
$$t_1(\theta) = -\frac{1}{2}\theta_1 y_1(\theta)\ ;\ t_2(\theta) = -\frac{1}{2}\theta_2 y_2(\theta)$$

The mechanism $\Gamma = (S_1, S_2, \dots, S_N, g(\cdot))$ implements the social choice function f with Bayesian Nash equilibrium strategies, if there exists Bayesian Nash equilibrium strategy vector $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ for the mechanism Γ, which results in such an equilibrium state, where the decision function is equal to the social choice function $g(s^*(\theta)) = f(\theta)$, for all $\theta \in \Theta$. The strategy $\frac{\theta_i}{2}$ is Bayesian Nash equilibrium strategy, and the results presented in the formula (21) is the equilibrium state resulting from the implementation of those strategies. Therefore, it can be stated, that first price sealed bid auction implements the social choice function which is presented in the formula (21).

It is obvious, that first price sealed bid auction is not incentive compatible. Particularly, the agent with θ_i type does not reveal his true type θ_i, but prefers to reveal another $\frac{\theta_i}{2}$ type, from which the expected utility is higher. According to incentive compatibility constraint the truthful revelation of types is optimal strategy, which is not true for the first price sealed bid auction.

According to the revelation principle, if there exists a mechanism $\Gamma = (S_1, S_2, \dots, S_N, g(\cdot))$, which implements the social choice function f in Bayesian Nash equilibrium, then the social choice function f is truthfully implementable in Bayesian Nash equilibrium. First price sealed bid auction implements the social choice function represented in the formula (21) in Bayesian Nash equilibrium, therefore, according to revelation principle, there exists another direct mechanism, which truthfully implements the social choice function represented in the formula (21).

The mechanism $\Gamma = (S_1, S_2, g(\cdot))$ is considered. The decision function $g(\cdot)$ is again $y_i = \{0,1\}$. If $y_i = 1$, then the product is bought by ith agent as $b_1 \geq b_0$, and if $y_i = 0$, then the product is not sold to ith agent as $b_1 < b_0$. In order to make the mechanism truthfully implementable the payment function should be adjusted, more particularly instead of $t_i = b_i y_i$, transfer function is adjusted to $t_i = \frac{b_i y_i}{2}$. The objective of the agent in the new mechanism is the following.

(18) $$\max((\theta_i - \frac{b_i(\theta_i)}{2})F(b_i(\theta_i)) = \max((\theta_i - \frac{b_i(\theta_i)}{2})\frac{b_i(\theta_i)-0}{1-0})$$

(19) $$(\theta_i b_i(\theta_i) - \frac{b_i(\theta_i)}{2} b_i(\theta_i))' = \left(\theta_i b_i(\theta_i) - \frac{b_i(\theta_i)^2}{2}\right)' = \theta_i - \frac{2b_i(\theta_i)}{2} = 0$$

(20) $$\theta_i - b_i(\theta_i) = 0 \leftrightarrow b_i(\theta_i) = \theta_i$$

It is obvious from the analysis above, that the adjusted mechanism truthfully implements the social choice function represented in the formula (21) with Bayesian Nash equilibrium strategy. In this case, according to the Bayesian Nash equilibrium strategy the agents having type θ_i place bids not equal to $\frac{\theta_i}{2}$, but reveal their true θ_i types instead. The designed mechanism is truthfully implementable and incentive compatible.

According to the objectives of the article the expected utilities of the agents and expected revenue of the seller should be calculated. As mentioned above, the types θ_i of the agents are random variables uniformly distributed in the range of [0,1]. In case of having type θ_i the ith agent places a bid $\frac{\theta_i}{2}$ and in case of winning that amount.

As the types are random variable with uniform distribution, then the expected utilities of agents can be calculated by integrating the utility function and the cumulative distribution function in the range of [0,1].

(21) $$\int_0^1 (\theta_i - \frac{\theta_i}{2})F(\theta_i)\,dx = \int_0^1 (\theta_i - \frac{\theta_i}{2})\theta_i = \int_0^1 \left(\theta_i^2 - \frac{\theta_i^2}{2}\right)$$

(22) $$\int_0^1 \left(\theta_i^2 - \frac{\theta_i^2}{2}\right) = \frac{\theta_i^3}{3} - \frac{\theta_i^3}{6} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$$

Based on formula (26) it can be deduced, that, if the types of the agents θ_i are random variables with uniform distribution in the range of [0,1], the expected utility of the ith agent from the first price sealed bid auction is 1/6.

The seller expects to receive from the winning agent an amount $\frac{\theta_i}{2}$:

(23) $$s = -(t_1(\theta) + t_2(\theta)) = \frac{1}{2}\theta_1 y_1(\theta) + \frac{1}{2}\theta_2 y_2(\theta)$$

As the types of the agents are random variables with uniform distribution, the expected revenue of the seller can be calculated by integrating the product of payment function $t_i(\theta)$ and the probability of winning in the range of [0,1]. And the expected revenue from the overall auction can be calculated by summing expected revenue from each agent.

(24) $$s = \int_0^1 t_1(\theta_1)F(\theta_1)dx + s = \int_0^1 t_2(\theta_2)F(\theta_2)dx$$

(25) $$s = \int_0^1 \frac{1}{2}\theta_1 F(\theta_1) + \int_0^1 \frac{1}{2}\theta_2 F(\theta_2)\,dx = \int_0^1 \frac{1}{2}\theta_1^2\,dx + \int_0^1 \frac{1}{2}\theta_2^2\,dx$$

(26) $$\int_0^1 \frac{1}{2}\theta_1^2 = \frac{1}{2}\frac{1^3}{3} = \frac{1}{6} \qquad \int_0^1 \frac{1}{2}\theta_2^2 = \frac{1}{2}\frac{1^3}{3} = \frac{1}{6}$$

(27) $$s = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

From the formula (31 it can be deduced, that, if the types of the agents θ_i are random variables uniformly distributed in the range of [0,1], the expected revenue of the seller from each agent is 1/6, and the overall expected revenue from auction is 1/3:

The conclusions derived regarding first price sealed bid auction consisting of two agents are summarized in the Table 1.

Table 1. *Conclusions regarding first price sealed bid auction*

First price sealed bid auction	
Number of agents	2
Bayesian Nash equilibrium	$\frac{\theta_i}{2}$
Equilibrium state	$y_1(\theta) = 0$ եթե $\theta_1 < \theta_2$ $y_2(\theta) = 1$ եթե $\theta_2 > \theta_2$ $y_1(\theta) = 0$ եթե $\theta_2 \le \theta_1$ $t_1(\theta) = -\frac{1}{2}\theta_1 y_1(\theta)$ $t_2(\theta) = -\frac{1}{2}\theta_2 y_2(\theta)$
Implemented social choice function	$y_1(\theta) = 0$ եթե $\theta_1 < \theta_2$ $y_2(\theta) = 1$ եթե $\theta_2 > \theta_2$ $y_1(\theta) = 0$ եթե $\theta_2 \le \theta_1$ $t_1(\theta) = -\frac{1}{2}\theta_1 y_1(\theta)$ $t_2(\theta) = -\frac{1}{2}\theta_2 y_2(\theta)$
The mechanism which truthfully implements the social choice function	$t_i = \frac{b_i y_i}{2}$
Probability of winning of an agent	θ_i
Expected utility of an agent	1/6 = 0.1667
Expected revenue of the seller	1/3 = 0.3333

Conclusions and recommendations

As a result of the analysis the following conclusions are derived:

- In first price sealed bid auction consisting of two agents the optimal or Bayesian Nash equilibrium strategy, which maximizes the expected utility of an agent is the strategy $\frac{\theta_i}{2}$.
- The Bayesian Nash equilibrium state of the mechanism, which describes first price sealed bid auction is represented in the formula (21), which also implements the social choice function.
- First price sealed bid auction is not truthfully implementable and incentive compatible. In order to truthfully implement the social choice function, a new mechanism is designed by adjusting the payment function $t_i = b_i y_i$ and replacing it with $t_i = \frac{b_i y_i}{2}$ function.
- The probability of winning of an agent in first price sealed bid auction consisting of two agents is θ_i.
- The expected revenue of an agent in first price sealed bid auction consisting of two agents is $\frac{1}{6}$.
- The expected revenue of the seller in first price sealed bid auction consisting of two agents is $\frac{1}{3}$.

Based on the analysis the following recommendation can be made:

- In the article the types are considered as random variables with uniform distribution. It is recommended to consider other statistical distribution and to observe their influence on the results of the model.
- First price sealed bid auction consisting of two agents has been analyzed in the article. It is recommended to give the general solution of the auction with N agents. It is also recommended to evaluate the effect of increase of the number of agents on the results of the auction.
- To make an empirical analysis based on statistical surveys or scientific experiments, giving an empirical assessment of the predictions made in the article.
- The analysis and resulting conclusions can be used to predict the results of first price sealed bid auction consisting of two agents.

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